Castle View School

Mathematics Reading Tasks

Year 10

2020/2021

Name……..

Contents

Leonardo of Pisa and the Golden Rectangle (AR Quiz 401010 EN)	**3.**
Maths Jobs in the Finance Sector – what do they involve? (AR Quiz 401008 EN)	**6.**
The History of Negative Numbers (AR Quiz 401042 EN)	**14.**
Eureka (AR Quiz 401060 EN)	**20.**
Egyptian Mathematics – Numbers and Numerals (AR Quiz 401070 EN)	**24.**
The impossible world of MC Escher (AR Quiz 401074 EN)	**28.**

Leonardo of Pisa and the Golden Rectangle

Age 7 to 16
Article by NRICH team
Published April 2003,February 2011.

Leonardo who? You know, Leonardo the son of Bonacci. The one who lived from about 1170 to 1250 in Pisa, the Italian city with the famous Leaning Tower.

Here is a picture of a statute of Leonardo that is in his home city. Do you recognise him now? (picture from the website of the School of Mathematics and Statistics University of St Andrews , Scotland)

Still puzzled?

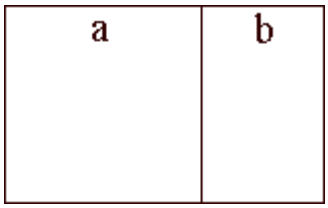
Well, Leonardo is better known as Fibonacci. His statue is not as well known as his number sequence.

There is an Fibonacci association and even a journal dedicated to the sequence. But there was more to Fibonacci than a number pattern and counting breeding rabbits. If you don't know about the rabbits you can find out more at Fibonacci's Three Wishes 2

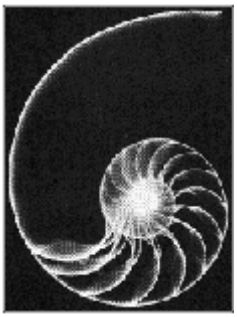
Fibonacci's number sequence (1,1,2,3,5,8,etc) has been discovered in some very interesting places. In branching plants, flower seeds and petal and leaf arrangements, on the outside of pineapples and inside apples as well as within pine cones. All involve the Fibonacci numbers - if you would like to discover how and why, Dr Ron Knott has the answers for you. You can do some Fibonacci puzzles if you visit the same site listed above.

If you investigate how the numbers of the sequence relate to each you will make another amazing discovery. Try dividing each number into the number that follows it, e.g. 2 into 3, 3 into 5, 5 into 8 and so on. Notice anything worth investigating further? Try finding the average of your answers. This is the type of strange phenomenon that intrigues mathematicians. If you have discovered a pattern to the answers, you have found an answer that is very close to what is sometimes known as the "golden mean".

Golden items are considered very valuable and quite rare. The "golden mean" has been a number used to investigate natural and man-made objects and the size of one part of the object to another part. When mathematicians examined what they considered to be perfect objects; a building like the Acropolis, a finely made and tuned violin, the human body, they discovered that the parts of the objects were in the same ratio as Fibonacci's numbers were to each other. They named this special relationship of the parts the "golden ratio".



This diagram gives you an idea of the proportions of a **Golden Rectangle** . It is divided into two pieces, and the ratio of the two parts (a to b) is the **Golden Ratio** . If you investigate even further you will find that the two parts together (a+b) is the same ratio to just the left part (a)!



If the **Golden Rectangle** is divided into two pieces, like in the diagram, the result is a square (a) and another rectangle (b) which is the same shape but a different size (or similar) to the original **Golden Rectangle** . There are more '**golden shapes** ' to be found in geometry and in nature. The nautilus shell or spiral, squares and triangles have all been investigated.

There are more '**golden shapes** ' to be found in geometry and in nature. The nautilus shell or spiral, squares and triangles have all been investigated.

The **Golden Rectangle** is said to be one of the most visually pleasing rectangular shapes. What do you think? Many artists think so and have used the shape within their artwork.

http://goldennumber.net

Whoever was the first to discover such intriguing evidence of geometry in nature, whether it was Fibonacci or not, must have been very excited about the discovery.

Maths Jobs in the Finance Sector – what do they involve?

Most people realise that maths plays a big part in finance and banking. In fact, around 20% of maths graduates enter the financial services sector. The majority of people will understand that maths is a useful skill for accountants, however in reality maths is used in a huge variety of ways in the finance sector, and accountancy only represents some of the jobs on offer. Even within accountancy there are enormous variations in what you could do, ranging from being a self employed accountant who works with small businesses to a forensic accountant who is investigating fraudulent activity.

What is in a job title?

One particular challenge for undergraduates is understanding some of the job titles which are found within the finance sector. Firstly, maths won't usually be mentioned in the job title and secondly it might describe a job you have never heard of before. Despite this, many jobs in finance are perfectly suited to maths graduates. This is why it is worth doing your research and making sure that you attend careers fairs and career presentations at your university.

Below we have listed some of the areas that maths might be used inside a bank or other financial institution. Hopefully it will help you understand that accountancy is only one part of the finance picture, and that accountancy itself is a very broad term, representing a hugely diverse range of activities.

Risk Management

Understanding risk is very important to banks. Banks want to make sure they haven't lent too much money to people who can't pay it back, and ultimately a bank needs to be able to remain solvent, even in the face of a potential future financial crisis.

Teams will work on quantifying risk using mathematical models, and those who work in this kind of role are often called 'quants'. They might be working to understand credit risk, liquidity or market risk and operations risk. One team will also look at economic stress testing of the bank's balance sheet – could the bank survive a serious shock to the economy? Another group might be looking at the risk-reward trade off on who to accept for a loan; and another section could be generating fraud detection algorithms which analyse spending patterns in order to detect money laundering activity.

Operations / Customer Banking

In this department, some of the teams will use Operational Research to help the bank make important business decisions, such as whether to open or close new branches. Other teams will use mathematical modelling to help the bank to be more effective at marketing and improve customer retention. For example, they will analyse customer behaviour and build models which can be used by behavioural economists to design products, and advertising and marketing campaigns.

Staff working in a high street branch of a bank are often incentivised to sell products via a system of rewards.

Teams in the head office will analyse these reward structures, checking that they are working and also that staff are not incentivised to sell the wrong products to the wrong customers, thereby avoiding compensation claims.

Products and Finance

In the products section of a bank, teams will use maths to perform effective product pricing, as well as looking at profitability and forecasting demand. In a finance department management accountants will work to perform calculations such as profit and loss forecasting, net present value calculations and so forth.

Technology

This department is data and programming based but is underpinned with the language of mathematics. This means that someone who has both programming and mathematics skills will be in high demand. Example projects could include producing algorithms for predicting income and spending patterns of customers and optimising finance within banks.

Financial Planning

Some people specialise in providing financial advice and investment management advice for individuals, business owners and families. This might include retirement planning, estate planning, life and income insurance, and mortgage planning.

Insurance Companies

Insurance is a huge industry within the financial sector. There are the standard insurance products which we all use such as home and contents insurance, as well as specialist insurance which deals with the aviation or marine sectors, or risks such as terrorism. Teams will analyse and price risk as well as setting policy terms and conditions.

Compliance

There is a lot of legislation surrounding the financial sector, which needs to be adhered to. Some people will become experts in areas such as compliance, data security, financial fraud prevention, client asset legislation or pensions legislation. This could be as a specialist in a company, as a consultant or by working for a body such as the FCA (Financial Conduct Authority).

Stockbroking

There are lots of different types of stockbrokers. You might be advising individuals or institutions on where to invest money, with many stockbrokers taking full charge of investment portfolios, rather than consulting with clients on a daily basis. This type of role can be very fast paced as the markets change. Having a strong mathematical background is important as you will have to absorb large amounts of financial information and have a good understanding of the figures involved.

Actuarial Profession

This can be a popular choice for maths graduates as actuaries work in all the above environments and more, including firms providing only specialist actuarial services. An actuary evaluates, manages and advises on financial risks, using statistical theory to assess the probability of an event occurring and its possible financial consequences. Actuaries often work in banking, corporate finance, mergers and acquisitions, investment management, general insurance plus healthcare, insurance product development and pensions scheme management.

Professional Qualifications

Many jobs within the financial sector involve studying for professional qualifications. This might seem like a burden after you have spent three or four years studying for a degree. However, this type of learning is very different from studying at university, as a lot of the knowledge will be directly relevant to the job. Many companies also offer paid time off for studying.

These qualifications can be really useful, not only because they will help you command a higher salary, but also because they could help you move jobs more easily in the future. For example if someone is a chartered management accountant in a big bank, they could make a transition to working in the charity sector or set up their own business in the future. The qualifications will give you a professional standing which will be really helpful throughout your career.

Other projects

Finance isn't just about traditional financial products such as mortgages, bank accounts and insurance. Working in finance is about understanding the real world, and making sense of it from a logical and financial perspective. Here are some examples of projects which might be looked at either by consultants working in the financial sector or by specialists working within a company.

- Creating and building models to predict which university students would drop out of university.
- Developing predictive models of how many coffees/teas/croissants/sandwiches will be sold by any shop by virtue of its location and prevailing weather.
- Selecting salesmen by predicting which traits will ultimately lead to superior performance.
- Building simulation models for Formula 1 cars which identify optimum strategies for when tyres should be changed – most Formula 1 teams will run over 1 million simulations during the course of a 2 hour race.
- Using physiological data to optimise what Formula 1 drivers eat, when they should sleep, when they need to fly to their next race and how to optimise their performance for the race.
- Working out the optimum level of stock by store to optimise how much stock is delivered and maintained by a store.
- Building predictive models of behaviour to work out when a trader is either about to make a judgement error or commit fraud.

- Work out the optimum number of police, nurses, paramedics and doctors that need to be available on different days of the week, and at different times of day to ensure that adequate staff are available to deal with peak demands.
- Car insurance – taking data from customers and telemetric systems to work out how safe or otherwise a driver is and then charging them an appropriate premium. Identifying when drivers should take breaks and for how long.
- Optimising the delivery route for a maintenance engineer or delivery driver, deciding which is quickest and uses the least amount of fuel.
- Working out whether a roulette table has a bias and whether that could be used by clients to beat the casino. Ensuring that the numbers in the National Lottery are entirely random or whether some bias has crept in.

  When you apply for a particular company, you should try and get a feel for the types of projects which they are involved with. Make sure you read their website, and talk to staff face to face at careers fairs. Many of these special projects might come under the umbrella of what is known as 'Operational Research' (OR). Some people will study a specialist masters degree or undergraduate degree in OR, whereas others will work on this type of project after a background of working in the financial or consultancy sectors.

Why choose the financial sector?

As a maths graduate there is a lot of choice when it comes to deciding on which career to pursue. What are some of the benefits to working in the financial sector?

- It can provide a stimulating and interesting work environment, with a lot of intellectual challenge.
- It allows you to gain professional qualifications which will benefit you for your whole career.
- It can offer the opportunity to live in different cities, and the chance to travel abroad if that is what you want.
- There is a lot of choice about your work setting – you could be working inside the headquarters of a large bank, or inside a small company or charity.
- Careers in this sector can be well paid.

Many undergraduates apply for internships, which allow them to find out if they are suited to a career in the financial sector. It is really worth doing your homework on where you want to be, as there are so many different types of roles to choose from. Many graduate schemes give you a chance to experience different departments within a company, allowing people to find out which area of the company they are most suited to.

This article was prepared with help from members of the IMA who work in the finance sector.

Image credits:

That was supposed to be going up, wasn't it? by Rafael Matsunaga@Flickr

Meeting by Vitaliy Kalyniuk@Flickr

Economy by Jason Kuffer@Flickr

The History of Negative Numbers

Age 11 to 18
Article by Leo Rogers

Published January 2008,February 2011.

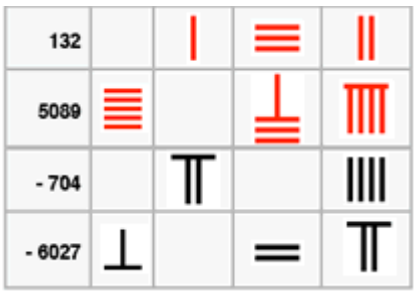
Although the first set of rules for dealing with negative numbers was stated in the 7th century by the Indian mathematician Brahmagupta, it is surprising that in 1758 the British mathematician Francis Maseres was claiming that negative numbers

"... darken the very whole doctrines of the equations and make dark of the things which are in their nature excessively obvious and simple" .

Maseres and his contemporary, William Friend took the view that negative numbers did not exist. However, other mathematicians around the same time had decided that negative numbers could be used as long as they had been eliminated during the calculations where they appeared.

It was not until the 19th century when British mathematicians like De Morgan, Peacock, and others, began to investigate the 'laws of arithmetic' in terms of logical definitions that the problem of negative numbers was finally sorted out.

However, there were references to negative numbers far earlier...



In 200 BCE the Chinese number rod system (see note1 below) represented positive numbers in Red and Negative numbers in black. An article describing this system can be found here . These were used for commercial and tax calculations where the black cancelled out the red. The amount sold was positive (because of receiving money) and the amount spent in purchasing something was negative (because of paying out); so a money balance was positive, and a deficit negative.

The concept also appeared in Astronomy where the ideas of 'strong' and 'weak' were used for approximating a number from above or below. For example approaching 5 from above means for example, starting with 5.2 you can find better approximations 5.1, 5.05, 5.025. Thus 5.025 was called a 'strong' approximation and a number like 4.9 'weak'. So 'strong' numbers were called positive and 'weak' numbers negative

In **India** , negative numbers did not appear until about 620 CE in the work of Brahmagupta (598 - 670) who used the ideas of 'fortunes' and 'debts' for positive and negative. By this time a system based on place-value was established in India, with zero being used in the Indian number sytem. Brahmagupta used a special sign for negatives and stated the rules for dealing with positive and negative quantities as follows:

A debt minus zero is a debt.
A fortune minus zero is a fortune.
Zero minus zero is a zero.
A debt subtracted from zero is a fortune.
A fortune subtracted from zero is a debt.
The product of zero multiplied by a debt or fortune is zero.
The product of zero multiplied by zero is zero.
The product or quotient of two fortunes is one fortune.
The product or quotient of two debts is one fortune.
The product or quotient of a debt and a fortune is a debt.
The product or quotient of a fortune and a debt is a debt.

The conflict between geometry and algebra

The ancient **Greeks** did not really address the problem of negative numbers, because their mathematics was founded on geometrical ideas. Lengths, areas, and volumes resulting from geometrical constructions necessarily all had to be positive. Their proofs consisted of logical arguments based on the idea of magnitude. Magnitudes were represented by a line or an area, and not by a number (like 4.3 metres or 26.5 cubic centimetres). In this way they could deal with 'awkward' numbers like square roots by representing them as a line. For example, you can draw the diagonal of a square without having to measure it (see note 2 below).

About 300 CE, the Alexandrian mathematician Diophantus (200 - c.284 CE) wrote his *Arithmetica* , a collection of problems where he developed a series of symbols to represent the 'unknown' in a problem, and powers of numbers. He dealt with what we now call linear and quadratic equations. In one problem Diophantus wrote the equivalent of 4 = 4x + 20 which would give a negative result, and he called this result 'absurd'.

In the 9th century in **Baghdad** Al - Khwarizmi (c.780 - c.850 CE) presented six standard forms for linear or quadratic equations and produced solutions using algebraic methods and geometrical diagrams. In his algebraic methodshe acknowledged that he derived ideas from the work of Brahmagupta and therefore was happy with the notion of negative numbers. However, his geometrical models (based on the work of Greek mathematicians) persuaded him that negative results were meaningless (how can you have a negative square?). In a separate treatise on the laws of inheritance, Al-Khwarizmi represents negative quantities as debts.

In the 10th century Abul -Wafa (940-998 CE) used negative numbers to represent a debt in his work on 'what is necessary from the science of arithmetic for scribes and businessmen'?. This seems to be the only place where negative numbers have been found in medieval Arabic mathematics. Abul-Wafa gives a general rule and gives a special case where subtraction of 5 from 3 gives a "debt" of 2. He then multiples this by 10 to obtain a "debt" of 20, which when added to a 'fortune' of 35 gives 15.

In the 12th century Al - Samawal (1130 - 1180) had produced an algebra where he stated that:

- if we subtract a positive number from an 'empty power', the same negative number remains,
- if we subtract the negative number from an 'empty power', the same positive number remains,
- the product of a negative number by a positive number is negative, and by a negative number is positive.

Negative numbers did not begin to appear in Europe until the 15th century when scholars began to study and translate the ancient texts that had been recovered from Islamic and Byzantine sources. This began a process of building on ideas that had gone before, and the major spur to the development in mathematics was the problem of solving quadratic and cubic equations.

As we have seen, practical applications of mathematics often motivate new ideas and the negative number concept was kept alive as a useful device by the Franciscan friar Luca Pacioli (1445 - 1517) in his *Summa* published in 1494, where he is credited with inventing double entry book-keeping.

Solving equations

The story of the solution of equations begins in Italy in the 16th century (see note 3 below). This story is full of intrigue and deception because methods of solution were kept secret. The issue which caused most consternation at the time was the meaning of $-1\sqrt{---}$*. In fact, Cardano (1501 - 1576) in his* Ars Magna *of 1545 had to solve a problem where* $-15\sqrt{----}$ *appeared.*
Cardano found a sensible answer (see note 4 below) by working through the algorithm, but he called these numbers 'ficticious' because not only did they disappear during the calculation, but they did not seem to have any real meaning. However, by 1572, the Italian engineer, Bombelli (1526 - 1572) had provided the correct rules for working with these 'imaginary' numbers(see note 5 below).

In the 17th and 18th century, while they might not have been comfortable with their 'meaning' many mathematicians were routinely working with negative and imaginary numbers in the theory of equations and in the development of the calculus.

The English mathematician, John Wallis (1616 - 1703) is credited with giving some meaning to negative numbers by inventing the number line, and in the early 18th century a controversy ensued between Leibniz, Johan Bernoulli, Euler and d'Alembert about whether $\log(-x)$ was the same as Log(x).

By the beginning of the 19th century Caspar Wessel (1745 - 1818) and Jean Argand (1768 - 1822) had produced different mathematical representations of 'imaginary'numbers, and around the same time Augustus De Morgan (1806 - 1871), George Peacock (1791 - 1858) William Hamilton (1805 - 1865) and others began to work on the 'logic'of arithmetic and algebra and a clearer definition of negative numbers, imaginary quantities, and the nature of the operations on them began to emerge.

Negative numbers and imaginaries are now built into the mathematical models of the physical world of science, engineering and the commercial world.

There are many applications of negative numbers today in banking, commodity markets, electrical engineering, and anywhere we use a frame of reference as in coordinate geometry, or relativity theory.

Pedagogical Note:

It seems that the problems that people had (and now have - see the Lottery incident) in understanding the use of negative numbers concerns:

- the difference between the *operation* of subtraction and the *object* (a negative number), since the same sign is used for both
- the language involved like 'minus minus 3' as opposed to 'subtract negative 3'
- separating the physical model or analogy (be it profit/loss or rise/fall in temperature or rotation/direction in the plane) from the rules of operating on the entities.

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William Frend
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Ifrah, G. (1998) The Universal History of Numbers . Harvill Press, London.

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Mactutor at St Andrews University http://www-history.mcs.st-and.ac.uk/history/ .

Eureka!

Article by Jenni Back and Liz Pumfrey

Published February 2011.

Have you ever heard anyone use this word before? It's actually Greek (meaning "I've found it!") and many people would associate it with a man called **Archimedes** . Here you'll be able to find out about Archimedes and in particular his Spiral.

In 322 BC, Alexander the Great built the city of Alexandria and soon afterwards, under the reign of Ptolemy, it became the capital of Egypt. To attract clever people to this city, Ptolemy set up a university in Alexandria, which was the first of its kind. It was at the University of Alexandria that nearly all the famous mathematicians of the time studied or taught.

Archimedes was one of these. He was born in about 287 BC in the city of Syracuse on the island of Sicily. We think that Archimedes visited Alexandria and studied there, as he seems to have had many friends who were also great mathematicians.

Archimedes on a Greek stamp

This image was taken from the following website:

http://www-groups.dcs.standrews.ac.uk/~history/Mathematicians/Archimedes.html

There are many stories about Archimedes' discoveries and inventions. When the Romans were attacking Syracuse, the King (who was also a good friend) asked Archimedes to help defend the city. He invented huge cranes, which could lift ships out of water; movable poles which dropped weights on enemy ships and powerful catapults.

Archimedes' Claw, a device which lifted ships into the air and rolled them over.

www.mcs.drexel.edu/~crorres/Archimedes/Claw/illustrations.html (unfortunately this link is no longer active)

Archimedes also developed complicated pulley systems to drag ships along. These war-time inventions gained Archimedes a lot of recognition and respect from the local people.

It was when Archimedes was in his bath that he jumped up shouting the famous "Eureka!" He had been trying to solve a problem for the King - how to find out whether the King's crown was pure gold or if some silver had been added. As he got into the bath, some water sloshed over the sides and this inspired Archimedes to try an experiment. He discovered that when the crown was put in a bowl of water, more water over-flowed than when the same weight of pure gold was put in. This meant that the crown could not be entirely pure gold. This led to the first law of hydrostatics. You can find out about this in more detail on the websites listed at the end.

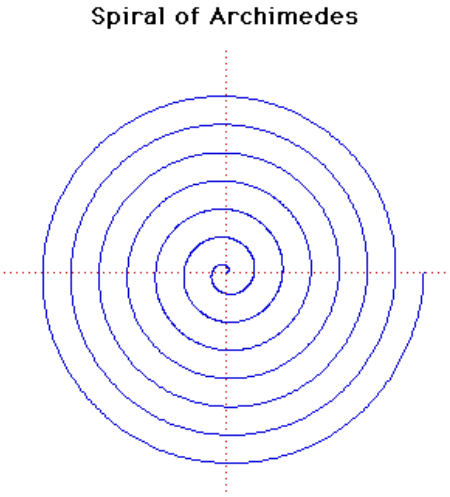
Although Archimedes became famous for his inventions during the war, he loved to study "pure" maths much more. He had a passion for geometry and he wrote many important papers on different topics. Here we shall concentrate on his spiral.

Archimedes' Spiral

A spiral is a special kind of curved path on a 2d (flat) surface. Spirals are described in maths by sets of equations and are given different names. Archimedes investigated many different types of spirals and one of these is now named after him.

The Archimedes' spiral is the path of a point that is travelling towards or away from a fixed origin at a **constant speed** and **constant velocity** . Velocity is a measure of speed in a particular direction.

This is what it looks like:



Used by permission

www-groups.dcs.st-andrews.ac.uk/~history/Curves/Spiral.html

As if all this wasn't enough, Archimedes investigated many other branches of geometry. One of his most famous proofs was the calculation of pi to between 3 10/71 and 3 1/7. (See Pi, AVery Special Number)

It is said that Archimedes used to get very absorbed in his work and would draw diagrams in sand, ashes from the fire and even on himself using bathing oils! One story tells that it was due to this total absorption in maths that Archimedes was killed. Apparently, he asked a Roman soldier to stand out of the way of a diagram he had sketched in the sand. The soldier ran a spear through his chest.

This Roman floor mosaic shows Archimedes' death

http://www-groups.dcs.st-andrews.ac.uk/~history/Mathematicians/Archimedes.html

Archimedes can be regarded as one of the most accomplished mathematicians in history. His discoveries certainly seem remarkable, considering how long ago he lived. We've only touched the surface here so if you'd like to read more about him, why not take a look at some of the websites listed below? Go on...

www-groups.dcs.st-andrews.ac.uk/~history/Mathematicians/Archimedes.html

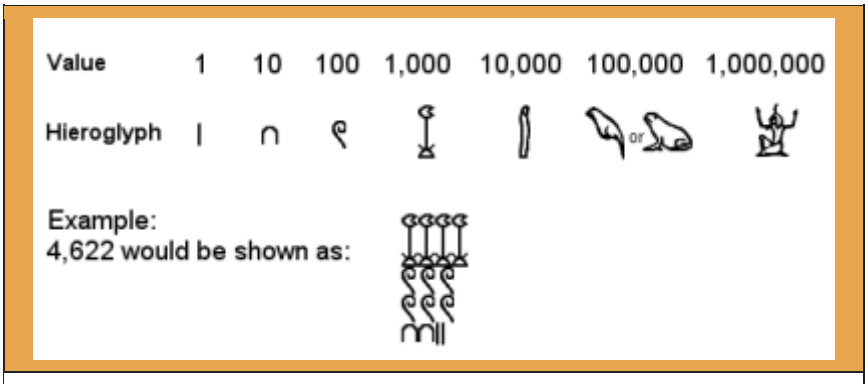
www-maths.mcs.st-andrews.ac.uk/~gmp/gmpANA.html

physics.weber.edu/carroll/Archimedes/theIndex.htm

There is also a lot of information about Archimedes in "An Introduction to the History of Mathematics" (4th edition) by Howard Eves published by Holt, Rinehart and Winston.

EGYPTIAN MATHEMATICS – NUMBERS & NUMERALS

The early Egyptians settled along the fertile Nile valley as early as about 6000 BCE, and they began to record the patterns of lunar phases and the seasons, both for agricultural and religious reasons.



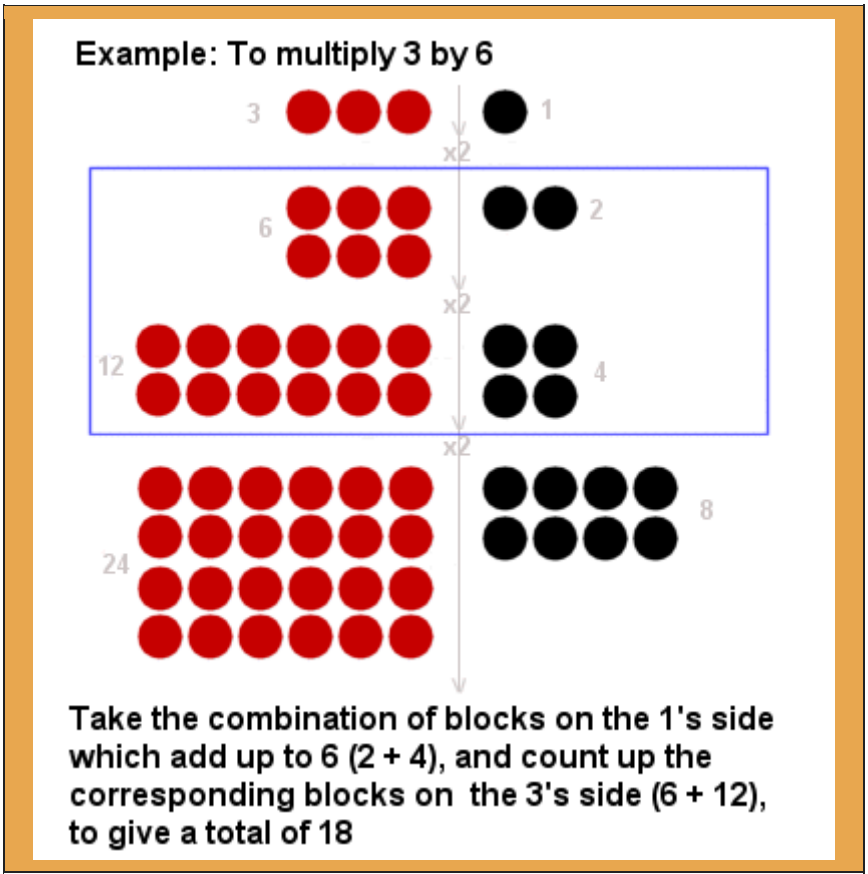
Ancient Egyptian hieroglyphic numerals

The Pharaoh's surveyors used measurements based on body parts (a palm was the width of the hand, a cubit the measurement from elbow to fingertips) to measure land and buildings very early in Egyptian history, and a decimal numeric system was developed based on our ten fingers. The oldest mathematical text from ancient Egypt discovered so far, though, is the Moscow Papyrus, which dates from the Egyptian Middle Kingdom around 2000 – 1800 BCE.

Ancient Egyptian Number System

It is thought that the Egyptians introduced the earliest fully-developed base 10 numeration system at least as early as 2700 BCE (and probably much early). Written numbers used a stroke for units, a heel-bone symbol for tens, a coil of rope for hundreds and a lotus plant for thousands, as well as other hieroglyphic symbols for higher powers of ten up to a million. However, there was no concept of place value, so larger numbers were rather unwieldy (although a million required just one character, a million minus one required fifty-four characters).

The Rhind Papyrus, dating from around 1650 BCE, is a kind of instruction manual in arithmetic and geometry, and it gives us explicit demonstrations of how multiplication and division was carried out at that time. It also contains evidence of other mathematical knowledge, including unit fractions, composite and prime numbers, arithmetic, geometric and harmonic means, and how to solve first order linear equations as well as arithmetic and geometric series. The Berlin Papyrus, which dates from around 1300 BCE, shows that ancient Egyptians could solve second-order algebraic (quadratic) equations.

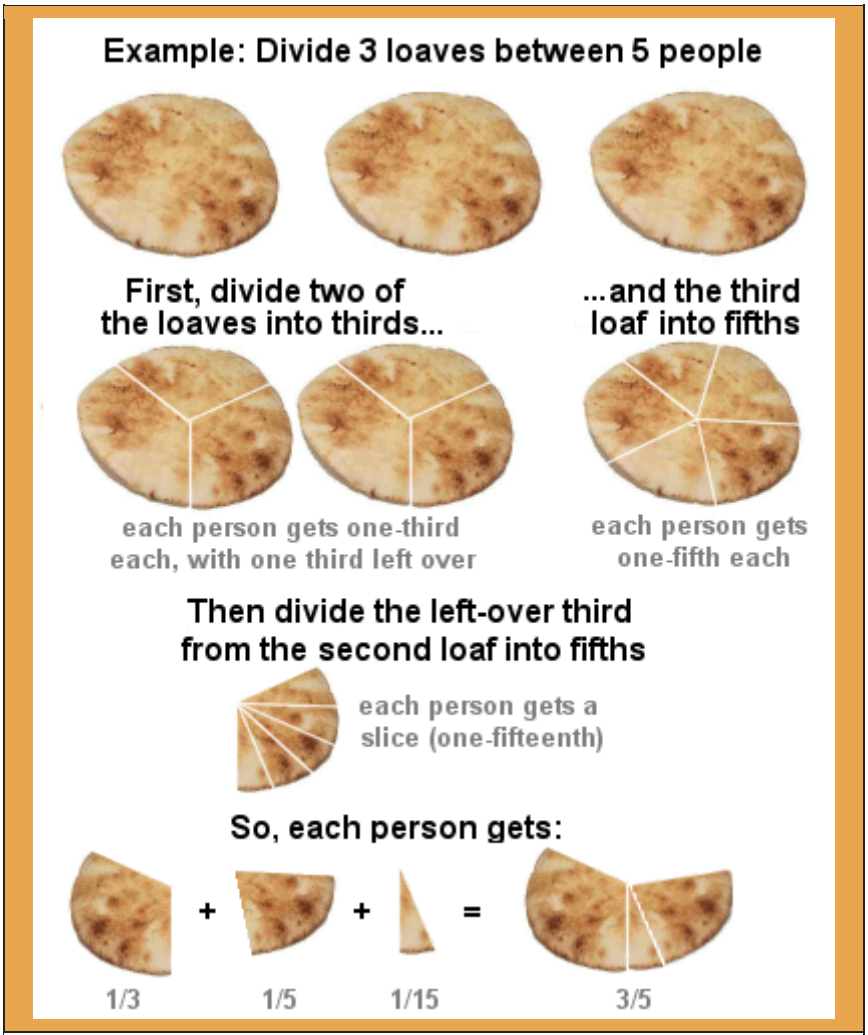


Ancient Egyptian method of multiplication

Multiplication, for example, was achieved by a process of repeated doubling of the number to be multiplied on one side and of one on the other, essentially a kind of multiplication of binary factors similar to that used by modern computers (see the example at right). These corresponding blocks of counters could then be used as a kind of multiplication reference table: first, the combination of powers of two which add up to the number to be multiplied by was isolated, and then the corresponding blocks of counters on the other side yielded the answer. This effectively made use of the concept of binary numbers, over 3,000 years before **Leibniz** introduced it into the west, and many more years before the development of the computer was to fully explore its potential.

Practical problems of trade and the market led to the development of a notation for fractions. The papyri which have come down to us demonstrate the use of unit fractions based on the symbol of the Eye of Horus, where each part of the eye represented a different fraction, each half of the previous one (i.e. half, quarter, eighth, sixteenth, thirty-second, sixty-fourth), so that the total was one-sixty-fourth short of a whole, the first known example of a geometric series.

Unit fractions could also be used for simple division sums. For example, if they needed to divide 3 loaves among 5 people, they would first divide two of the loaves into thirds and the third loaf into fifths, then they would divide the left over third from the second loaf into five pieces. Thus, each person would receive one-third plus one-fifth plus one-fifteenth (which totals three-fifths, as we would expect).



Ancient Egyptian method of division

The Egyptians approximated the area of a circle by using shapes whose area they did know. They observed that the area of a circle of diameter 9 units, for example, was very close to the area of a square with sides of 8 units, so that the area of circles of other diameters could be obtained by multiplying the diameter by $^{8}/_{9}$ and then squaring it. This gives an effective approximation of π accurate to within less than one percent.

The pyramids themselves are another indication of the sophistication of Egyptian mathematics. Setting aside claims that the pyramids are first known structures to observe the golden ratio of 1 : 1.618 (which may have occurred for purely aesthetic, and not mathematical, reasons), there is certainly evidence that they knew the formula for the volume of a pyramid – $^{1}/_{3}$ times the height times the length times the width – as well as of a truncated or clipped pyramid.

They were also aware, long before **Pythagoras**, of the rule that a triangle with sides 3, 4 and 5 units yields a perfect right angle, and Egyptian builders used ropes knotted at intervals of 3, 4 and 5 units in order to ensure exact right angles for their stonework (in fact, the 3-4-5 right triangle is often called “Egyptian”).

The impossible world of MC Escher

He was dismissed by the art world and venerated by mathematicians … he rejected both Mick Jagger's and Stanley Kubrick's attempts to schmooze him. So who was the mysterious MC Escher, master of illusion?

Day and Night, 1938, by MC Escher. Images courtesy of Collection Gemeentemuseum Den Haag/the MC Escher Company

Steven Poole
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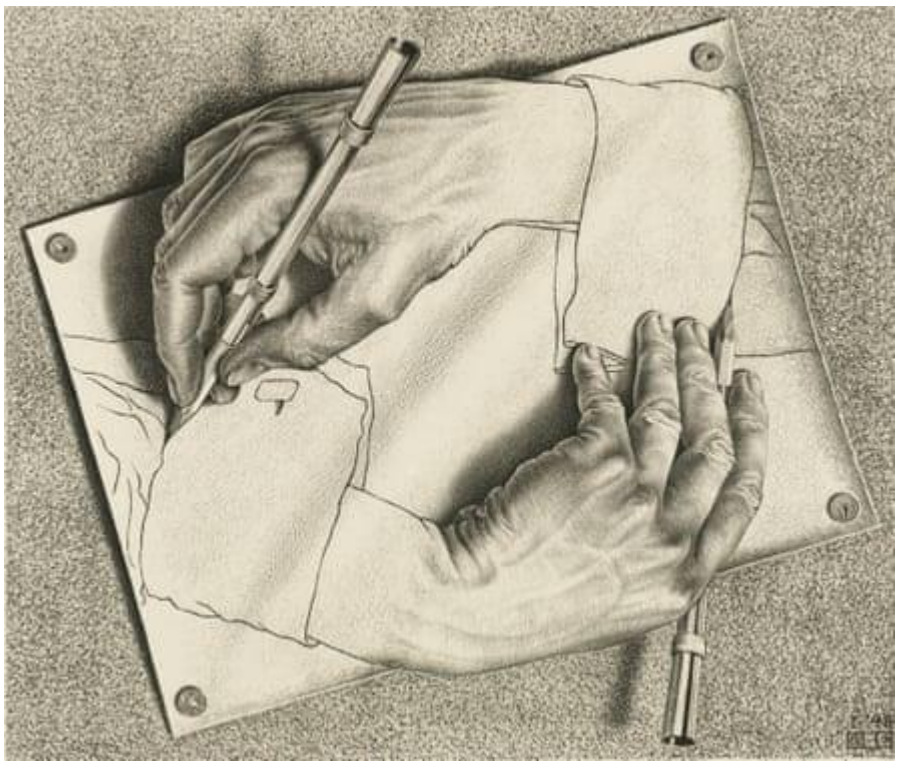
The artist who created some of the most memorable images of the 20th century was never fully embraced by the art world. There is just one work by Maurits Cornelis Escher in all of Britain's galleries and museums, and it was not until his 70th birthday that the first full retrospective exhibition took place in his native Netherlands. Escher was admired mainly by mathematicians and scientists, and found global fame only when he came to be considered a pioneer of psychedelic art by the hippy counterculture of the 1960s. His prints adorn albums by Mott the Hoople and the Scaffold, and he was courted unsuccessfully by Mick Jagger for an album cover and by Stanley Kubrick for help transforming what became *2001: A Space Odyssey* into a "fourth-dimensional film".

But Escher did not belong to any movement. In a 1969 letter to a friend, he observed testily that “the hippies of San Francisco continue to print my work illegally”. (Many of his letters are reproduced in the standard reference book, *Escher: The Complete Graphic Work*, edited by JL Locher, which includes a full biography and analytical essays by Escher and others.) He had been sent a catalogue for a California “Free University” that contains “three reproductions of my prints alternating with photographs of seductive naked girls”. This would have seemed distasteful to the rather formal Escher, who bridled when Jagger addressed him by his first name in a fan letter. According to Patrick Elliott’s catalogue essay, “Escher and Britain”, for the new exhibition at the Scottish National Gallery of Modern Art, *The Amazing World of MC Escher*, the artist replied to the musician’s assistant: “Please tell Mr Jagger I am not Maurits to him.”
To his family and childhood friends Maurits was affectionately known as Mauk. He was born to George and Sara Escher in 1898 in Leeuwarden. The youngest of his civil-engineer father’s five sons (two from a previous marriage), Mauk was a sickly child who was interested in carpentry and took music lessons, but failed his final school exams, except for mathematics. His father noted fondly in his diary that the young man consoled himself “by drawing and making a linocut of a sunflower”.



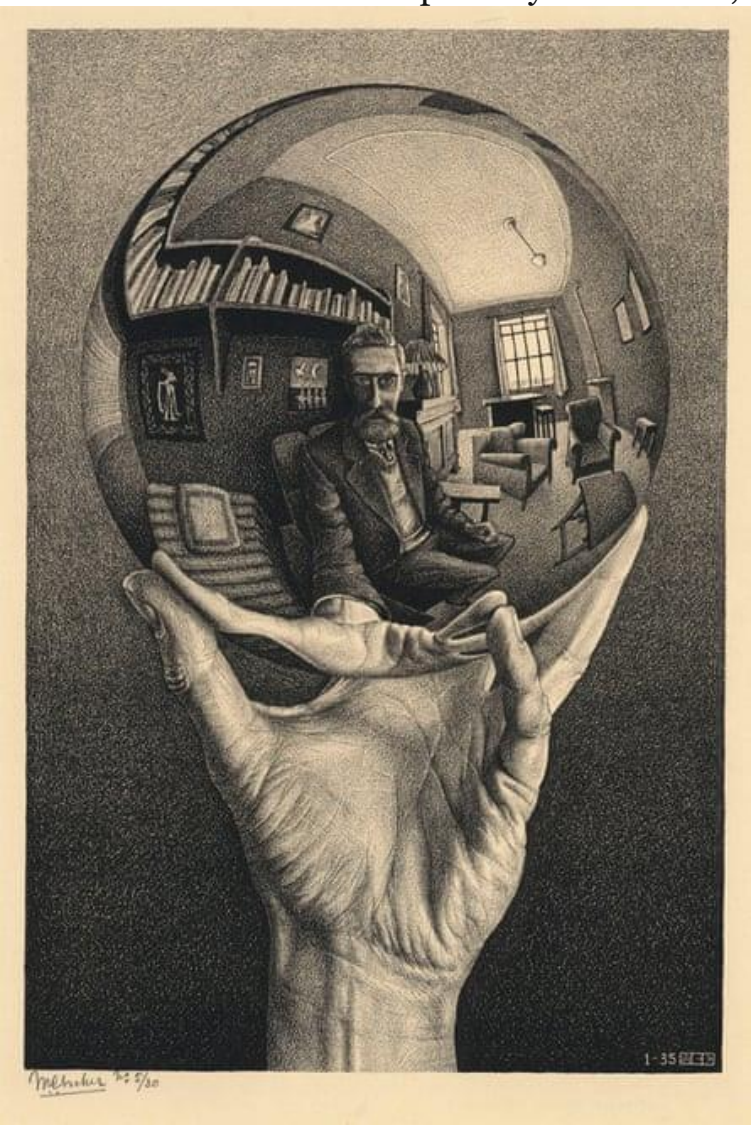
Drawing Hands, 1948, by MC Escher

Escher then studied for a few years at the School for Architecture and Decorative Arts in Haarlem, but he abandoned architecture to try to carve out a career as a graphic artist. It quickly went well. By the end of the 1920s, during which he had travelled extensively in Italy and Spain, and met and married his wife, Jetta, Escher was exhibiting his work regularly in Holland, and, in 1934, he won his first American exhibition prize. But it was only two years later that Escher really became Escher. That year he went to the Alhambra Palace in Granada, Spain, and carefully copied some of its geometric tiling. His work gradually became less observational and more formally inventive. As Escher later explained, it also helped that the architecture and landscape of his successive homes in Switzerland, Belgium and Netherlands were so boring: he "felt compelled to withdraw from the more or less direct and true-to-life illustrating of my surroundings", embracing what he called his "inner visions".

Those visions fed what would become Escher's most celebrated works. In 1948, he made *Drawing Hands*, the image of two hands, each drawing the other with a pencil. It is a neat depiction of one of Escher's enduring fascinations: the contrast between the two-dimensional flatness of a sheet of paper and the illusion of three-dimensional volume that can be created with certain marks. In *Drawing Hands*, space and the flat plane coexist, each born from and returning to the other, the black magic of the artistic illusion made creepily manifest. The following, from a later Escher essay, could easily serve as a gloss on this image:
The artist still has the feeling that moving his pencil over the paper is a kind of magic art. It is not he who determines his shapes; it seems rather that the stupid flat shape at which he painstakingly toils has its own will (or lack of will), that it is this shape which decides or hinders the movement of the drawing hand, as though the artist were a spiritualist medium.

Escher's lifelong subject, in a way, was the dramatised artificiality of the created image. (The art historian EH Gombrich wrote that Escher's work "presents so many interesting comments on the puzzles of representation".) Of his 1945 picture *Balcony*, with its weird bulging central distortion, Escher commented: "Surely it is a bit absurd to draw a few lines and then claim 'This is a house.'" The theme of *Balcony*, he said, was "this odd situation".

The situations only became odder. Escher methodically pushed representative techniques to their limits. Some earlier images suggest a particular interest in perspective, for instance the “bird’s-eye view” of his *Tower of Babel* (1928) – which in retrospect seems like an unparadoxical rehearsal for his later adventures in impossible architecture – or a forest of columns in a colonnade for *Nocturnal Rome* (1934). A later image, *Depth* (1955), is an entirely fictional investigation into the formal possibilities of perspective: an array of what look like monstrous robotic fish-aeroplanes, receding implicitly into infinite space. (In a letter, Escher explained the carefully thought-out features that contribute to the depth effect, including “Rhythmic positioning of each fish at the intersections of a cubic threefold-rotation-point system”. Well, naturally.)



Hand with a Reflecting Sphere, 1935, by MC Escher

In the late 1930s, Escher also became obsessed by the "regular division of the plane", in which shapes (often fish, lizards or birds) are tiled across a flat plane in such a way that the spaces between them make other, recognisable shapes. (This technique was directly inspired by the Alhambra.) *Day and Night* (1938) features black and white bird forms arranged in this way over a chequerboard countryside. In many of these images the distinction between foreground and background is obliterated: the viewer can choose to see one or other set of shapes as foreground at will.

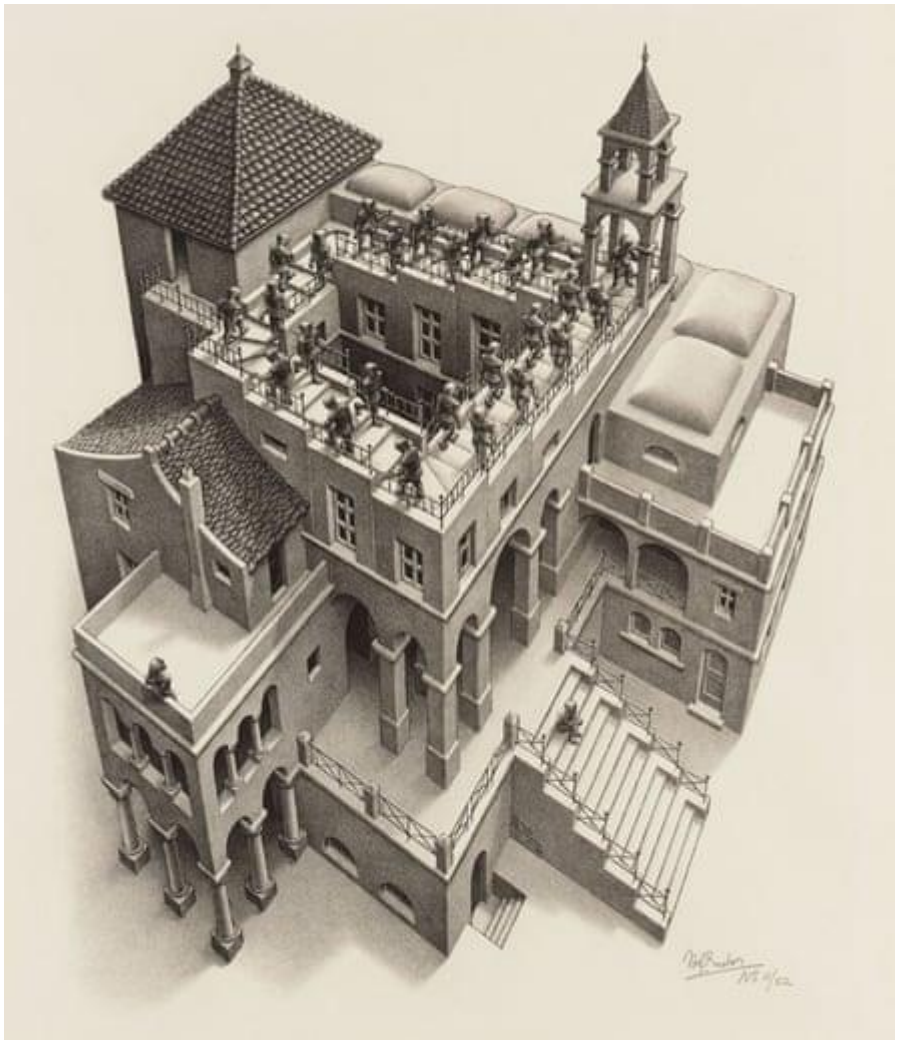
But if Escher's work had been nothing more than a pedantic meta-artistic commentary on the plasticity of techniques, it would by now have been forgotten. He gave a revealing lecture in 1953 that distinguished between "feeling people" – artists who concentrate on the human form – and "thinking people", artists like himself who are "reality enthusiasts", interested in "the language of matter, space and the universe". Escher's greatest pictures are not simply geometric exercises; they marry formal astonishment with a vivid and idiosyncratic vision.

Take *House of Stairs* (1951), with its nightmarish interior (inspired by Escher's own school's stairways) and its pseudo-human-faced articulated centipedes crawling through its architectural phantasmagoria. (Escher invented those creatures, he explained wryly, "as a result of dissatisfaction concerning nature's lack of any wheel-shaped living creatures endowed with the power of propulsion by means of rolling themselves up".)

Or *Belvedere* (1958), with its impossible ladder and a collection of those jesters, knaves and contemplators who would come to populate Escher's most extraordinary invented places. (The long-gowned woman in *Belvedere* is copied directly from *The Garden of Earthly Delights* by Hieronymus Bosch.)

Most dazzling, perhaps, is the celebrated *Ascending and Descending* (1960), with its two ranks of human figures trudging forever upwards and eternally downwards respectively on an impossible four-sided eternal staircase. It is the most recognisable of Escher's "impossible objects" images, which were inspired by the British mathematician Roger Penrose and his father, the geneticist Lionel Penrose. Fascinated by *House of Stairs*, the Penroses published a paper in 1956 in the British Journal of Psychology entitled "Impossible Objects: A Special Type of Visual Illusion". Receiving an offprint a few years later, Escher wrote to Lionel expressing his admiration for the "continuous flights of steps" in the paper, and enclosing a print of *Ascending and Descending*. (The paper also included the "tri-bar" or Penrose triangle, which is constructed impossibly from three 90-degree angles: in 1961 Escher built his never-ending *Waterfall* using three of them.)

The mathematical trickery in *Ascending and Descending*'s staircase is not the subject of the image. Escher was never a surrealist. But in this picture, it becomes clear that he was a kind of existentialist. He had long admired Dostoyevsky and Camus, and in a letter to a friend while he was working on *Ascending and Descending* he explained: "That staircase is a rather sad, pessimistic subject, as well as being very profound and absurd. With similar questions on his lips, our own Albert Camus has just smashed into a tree in his friend's car and killed himself. An absurd death, which had rather an effect on me. Yes, yes, we climb up and up, we imagine we are ascending; every step is about 10 inches high, terribly tiring – and where does it all get us? Nowhere."



Ascending and Descending, 1960, by MC Escher

This dreamscape of futility is perfected by the two figures who are not on the eternal staircase. One gazes up at his condemned fellows from a side terrace; one sits glumly on the lower stairs. “Two recalcitrant individuals refuse, for the time being, to take any part in this exercise,” Escher commented. “They have no use for it at all but no doubt sooner or later they will be brought to see the error of their nonconformity.” Escher’s art at its best, then, is not just surprising but also surprisingly readable, putting him in the company of the great allegorical printmakers such as Albrecht Dürer.

Since Escher’s death in 1972, his most famous images have become ubiquitous. New fuel for his popular cult was provided by Douglas Hofstadter’s interdisciplinary fantasia of a book, *Gödel, Escher, Bach* (1979), which seduced generations of curious students in the following decades. (Escher adored Bach.) Fittingly, given the artist’s mathematical playfulness, some of the richest tributes to his work in modern times have come in the world of video games. In the beautiful *Echochrome* (2008), players set out to free an eternally walking human from a succession of Escherian landscapes by rotating the point of view until the “trick” of perspective locks into place.
In a 1963 lecture on “the impossible”, Escher declared: “If you want to express something impossible, you must keep to certain rules. The element of mystery to which you want to draw attention should be surrounded and veiled by a quite obvious, readily recognisable commonness.” This is arguably as true of fiction or music as it is of Escher’s brand of geometric sorcery. And it also, in a way, sums up the genius of Escher himself, an orderly man who made inexhaustibly extraordinary things.

• *The Amazing World of MC Escher* is at the Scottish National Gallery of Modern Art, Edinburgh, from 27 June to 27 September. nationalgalleries.org.